Operating Room Managers' Use of Integer Programming for Assigning Block Time to Surgical Groups: A Case Study

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A common problem at hospitals with fixed amounts of available operating room (OR) time (i.e., "block time") is determining an equitable method of distributing time to surgical groups. Typically, facilities determine a surgical group's share of available block time using formulas based on OR utilization, contribution margin, or some other performance metric. Once each group's share of time has been calculated, a method must be found for fitting each group's allocated OR time into the surgical master schedule. This involves assigning specific ORs on specific days of the week to specific surgical groups, usually with the objective of ensuring that the time assigned to each group is close to its target share. Unfortunately, the target allocated to a group is rarely expressible as a multiple of whole blocks. In this paper, we describe a hospital's experience using the mathematical technique of integer programming to solve the problem of developing a consistent schedule that minimizes the shortfall between each group's target and actual assignment of OR time. Schedule accuracy, the sum over all surgical groups of shortfalls divided by the total time available on the schedule, was 99.7% (SD 0.1%, $n = 11$). Simulations show the algorithm's accuracy can exceed 97% with ≥4 ORs. The method is a systematic and successful way to assign OR blocks to surgeons.

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A variety of hospitals receive funding as annual global payments for providing care (e.g., those in Canada or hospitals in the United States' Department of Veterans Affairs). Once a budget has been selected for the hospital, it is the hospital administrators' responsibilities to not exceed the budgeted amount.

Administrators at such hospitals typically focus attention on staffed hours of operating room (OR) time because staffed OR hours represent a large percentage of the total perioperative budget (1,2) and because access to OR time creates demand for costly items such as implants. When hospital administrators decide to increase or reduce the OR budget, responsibility rests with the OR manager to decide which surgical groups will have their allocated OR time increased or reduced. If surgeons are paid on a fee-for-service basis, as they are in most Canadian hospitals, changes to allocated OR hours directly affect professional income. The allocation process is often fraught with political ramifications, in part because of its economic impact on surgeons.

At hospitals with a fixed annual budget, surgeons are typically allocated block time into which they are able to book elective cases. Block booking rules permit an elective case to be scheduled only if it can reasonably be expected to be completed during the regularly scheduled hours of OR time (i.e., within block time) (3). However, the focus of this article is not on methods for calculating appropriate block time allocations, which can be done based on total hours of cases per allocated block (i.e., utilization), hospital costs per allocated block, previous number of allocated hours, contribution margin per allocated block, or some other metric (4). In this study we examine how to assign specific ORs on specific days of the week to specific surgical groups under the assumption that the target number of hours of OR time to be allocated to each surgical group has already been chosen (5).

Specifically, this study focuses on a methodology for assigning whole blocks of time to surgical groups (5). This can be difficult, both practically and theoretically, for two reasons. First, the target allocation to a group may not be expressible as a multiple of whole blocks. For example, if it is determined that a cardiac surgery group should be allocated 35.96 h of OR time per week and the surgical suite utilizes 8-h blocks, the

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group's actual allocation will be either 32 h or 40 h. Second, even if target allocations are multiples of whole blocks, the supply of staffed ORs and/or specialty equipment may restrict the actual number of hours that can be assigned to a group. For example, suppose that with an increase in the surgical suite's staff budget, the cardiac surgery group's target allocation increases to 48 h. If the surgical suite only has one OR with the equipment for cardiac surgery, and the budget limits staffing to an 8-h workday, then the group's actual allocation cannot exceed 40 h per week.

In this case study, we describe an application of the mathematical technique of integer programming (6,7) to the generation of block assignments at a surgical suite with a prespecified total number of hours of OR time. The approach incorporates restrictions on both specialized equipment and personnel and can be used at a variety of surgical suites because the target allocations of OR time used in the model can be based on utilization, contribution margin, cost, previous number of allocated hours, or some politically tenable combination of these factors.

Description of Integer Linear Programming

Integer programming is a form of mathematical programming. Mathematical programming is an optimization technique in which the objective is to find the minimum (or maximum) of a function comprised of a set of decision variables. The values of the decision variables must satisfy a set of restrictions, or constraints, including restrictions on the sign of the decision variables. If the objective function is linear and all of the constraints are linear, the mathematical program is known as a linear program. Furthermore, if the decision variables are all restricted to be integer-valued, the linear program is known as an integer program. The application of integer programming techniques for scheduling problems is well-established (7).

Description of a Master Surgical Schedule

The master surgical schedule is a cyclic timetable that defines the number and type of ORs available at a facility, the hours that ORs will be open, and the surgical groups or surgeons who are to be given priority for the OR time. The master surgical schedule, under a block scheduling methodology, assigns a fixed amount of OR time on a given day and time to a particular surgeon or group. Figure 1 shows a master surgical schedule with a time horizon of 1 wk.

At Mt. Sinai Hospital in Toronto, a new master surgical schedule is created whenever the total amount of OR time changes (e.g., when a different number of staffed ORs and/or a different duration of staffed hours are available). This can occur not only as a response to variations in the availability of staffed OR hours occurring as a result of annual changes in funding but also as a response to operational issues such as seasonal fluctuations in demand and staff availability during the summer and Christmas seasons. Consequently, in any given year, OR managers may produce several different master surgical schedules.

Creating a Master Surgical Schedule

The gross number of OR hours to be allocated is typically a function of the budget provided by the hospital for OR nursing. At some hospitals the budget would also include funding for nurse anesthetists. However, the hospital in this case does not employ nurse anesthetists. The OR manager develops a small number of alternative schedule arrangements that meet the gross number of hours and are feasible in terms of the nurses' contracts and/or other collective bargaining agreements. At this point, the schedule is simply a template indicating the number of ORs that will be staffed each day of the week and the hours that each of these ORs will be open. The template is simply the master surgical schedule in Figure 1 with all of the groups deleted.

The template used to create the master surgical schedule in Figure 1 has a number of features chosen for ease of development. The template defines a consistent number of staffed ORs each workday. There are the same numbers of 7.5-h, 8-h, and 9-h ORs each day. On Fridays the nursing staff has an hour-long "in-service" education meeting, so cases start an hour later. Because this hospital does not provide trauma care, block time is not specifically provided for emergency surgery; however, other hospitals could include such time.

Once a template for the master surgical schedule has been developed, each available block must be assigned to a surgical group and/or surgeon. Only one surgical group is assigned to each OR on any day. Splitting ORs is considered undesirable because the surgical group working in the morning may run late and cut into the time assigned to the afternoon group. Split blocks result in inaccurate forecasts of appropriate OR time allocations, case cancellations, and decreased surgeon and patient satisfaction. The master surgical schedule must also respect bounds on the minimum and maximum number of ORs that can be assigned to a surgical group on any given day. These limits can arise from equipment restrictions and/or surgeon availability limitations.

Two objectives are used to evaluate any new master surgical schedule. The first objective is ensuring that the total hours of OR time actually allocated (i.e., assigned) to each surgical group is as close as possible to a target allocated number of hours. Integer programming (6,7) is used to minimize the under supply of OR hours allotted

	Main 1	Main 2	Main 3	Main 4	Main 5	Main 6	Main 7	Main 8	Main 9	Main 10	OPS 1	OPS 2
Mon	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-17:00	Gyne 08:00-17:00	Surg 08:00-15:30	Surg, Otol[1] 08:00-15:30	Gyne 08:00-15:30	Opht 08:00-15:30	Not staffed	Not staffed	Oral 08:00-16:00	Gyne 08:00-15:30
Tue	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-15:30	Otol 08:00-15:30	Gyne 08:00-15:30	Oral, Opht[2] 08:00-15:30	Not staffed	Not staffed	Gyne 08:00-15:30	Gyne 08:00-16:00
Wed	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-17:00	Surg 08:00-15:30	Otol 08:00-15:30	Gyne 08:00-15:30	Gyne 08:00-15:30	Not staffed	Not staffed	Gyne 08:00-16:00	Opht 08:00-15:30
Thu	Surg 08:00-17:00	Surg 08:00-17:00	Gyne 08:00-17:00	Gyne 08:00-17:00	Surg 08:00-15:30	Gyne 08:00-15:30	Open 08:00-15:30	Opht 08:00-15:30	Not staffed	Not staffed	Gyne 08:00-16:00	Opht 08:00-15:30
Fri	Surg 09:00-17:00	Surg 09:00-17:00	Surg 09:00-17:00	Surg 09:00-17:00	Surg 09:00-15:30	Otol 09:00-15:30	Gyne 09:00-15:30	Opht 09:00-15:30	Not staffed	Not staffed	Oral 09:00-15:30	Gyne 09:00-16:00

1 Surg weeks 1 & 2; Otol weeks 3, 4 & 5
2 Oral weeks 1, 2 & 3; Opht weeks 4 & 5

Figure 1. Example of a master surgical schedule used at Mt. Sinai Hospital in Toronto. There are five surgical departments: Surgery (Surg), Gynecology (Gyne), Ophthalmology (Opht), Otolaryngology (Otol), and Oral Surgery (Oral). Any department can book a case into the "open," first-come first-served OR time on Thursday in OR #7. If more than one department is listed for an OR, then it is assigned to different departments on different weeks of the month. In particular, OR #6 is assigned to the Department of Surgery for the 1st and 2nd Mondays of the month, and Otolaryngology for the others. Operating room #8 is assigned to the Department of Oral Surgery for the 1st, 2nd, and 3rd Tuesdays of the month, and Ophthalmology otherwise.

to a surgical group relative to its targeted OR hours, while allocating whole blocks of time to groups (see Appendix). However, only rarely will the actual hours of OR time assigned to each surgical group precisely match the targeted OR hours for each group.

The second objective is to create a schedule that changes little from week to week. Although it would be ideal for the master surgical schedule to be identical week to week, this would frequently result in large differences between targeted and actual OR hours. However, for the targeted and actual OR hours to match precisely, the master surgical schedule might need to be different every week of the year.

A "good" master surgical schedule balances the competing objectives of a perfect allocation and a consistent schedule. The details of a process designed to achieve this are given in the Appendix.

Application at the Surgical Suite of Interest

At Mt. Sinai, the contentious issue among surgeons is generally not the magnitude of the change in OR time, which is a function of the amount of money made available by the provincial government to the hospital, but its equitable distribution among the five surgical groups. Although there are many ways to define "equitable," in the context of this problem an allocation is considered equitable if the proportion of total time allocated to a group before a schedule revision is the same as the proportion allocated after the schedule has been revised. For example, if the total staffed OR hours are decreased by 10%, the hours allocated to each of the five surgical groups should also decrease by precisely 10%.

Before implementation of the integer programming approach described in the Appendix (4), the OR nurse manager at Mt. Sinai Hospital used graph paper and a pencil with a large eraser to fill the various slots on the blank template. Changes in the number and duration of ORs available and constraints on specialty equipment were incorporated into the master surgical schedule by trial and error. A draft schedule derived from this manual process was circulated to the surgical groups for comments and suggestions for revision. Several iterations of this process were often necessary because revisions to accommodate each surgical group's requests affected the other groups. A final master surgical schedule was eventually created through negotiation.

Since implementation of the integer programming approach, the hospital has created the master surgical schedule using the method outlined in the Appendix. The algorithm was programmed in Microsoft Excel 2000 (Redmond, WA) and calls an add-in solver produced by Frontline Systems (Version 3.5, Incline Village, NV) for use with Excel. An Excel Visual Basic 6.0 user interface allows users to enter data, change model parameters, execute the algorithm, and view results such as the percentage of time lost or gained by each surgical group (Fig. 2).

	Surgery	Open	Gynecology	Ophthalmology	Oral Surgery	Otolaryngology	Total
Original block time (hours)	208.5	6.0	129.5	43.5	22.0	29.0	438.5
Original block time (% of total)	47.5%	1.4%	29.5%	9.9%	5.0%	6.6%	100.0%
Target block time (hours)	189.0	5.4	117.4	39.4	19.9	26.3	397.5
Allocated block time (hours)	189.0	7.5	117.0	38.8	19.7	25.5	397.5
Allocated block time (% of total)	47.5%	1.9%	29.4%	9.8%	5.0%	6.4%	100.0%
Difference between targeted and allocated block time (hours)	0.0	2.1	-0.4	-0.6	-0.3	-0.7	0.0
Difference between allocated and original percentage allocations (%)	0.0%	0.5%	-0.1%	-0.2%	-0.1%	-0.2%	0.0%
Absolute under supply of block time (hours)	0.0	0.0	0.4	0.6	0.3	0.7	2.0
Proportional under supply of block time (%)	0.0%	0.0%	0.1%	0.2%	0.1%	0.2%	0.5%

Figure 2. Total and percentage of operating room time lost or gained by each surgical group is calculated by the computer program for presentation to all surgical groups. In this example, before reallocation, the Department of Surgery held 208.5 h of OR time or 47.5% of the 438.5 h available each week. The target allocation of 189.0 h indicates the amount of time the Department of Surgery should be allotted to ensure it maintains a 47.5% share of available OR time under a schedule with 397.5 h available per week. In the actual allocation given in the table, the Department of Surgery received 189.0 h or 47.5% of the total hours available.

The interface also provides users with a means of adding, deleting, or modifying the constraints included in the model. Model constraints allow users to specify the minimum and maximum number and type of rooms that may be assigned to a department, or group of departments, for a given day, or over a group of days. This mechanism for entering constraints provides users with the ability to model a variety of situations related to equipment, staff availability, surgeon preferences, or other operational restrictions. For example, at Mt. Sinai, ophthalmologic surgery is constrained by the availability of specialized equipment. To model this situation, a constraint is included for each weekday that limits the number of rooms that can be assigned to the Department of Ophthalmology. Other constraints, such as a requirement that no more than five General Surgery rooms can be staffed on any day, are entered in a similar fashion. This approach to constraint definition provides users with the flexibility to create realistic models that are applicable under a wide variety of circumstances.

Results at the Surgical Suite of Interest

The program has been used to create all master schedules adopted at Mt. Sinai since January 1999. Eleven schedules have been developed using this methodology in response to changing budgets, variations in staff and surgeon availability, or seasonal fluctuations in demand. Operational schedules developed have included different total numbers of ORs available (range, 8–12 ORs), durations of blocks (range, 6.5–9.5 h), availability of an emergency inpatient surgery block (range, 0–1), and problem specific constraints (e.g., the minimum and maximum number of ORs, of a particular type, that may be assigned to a surgical group on a given day or over the course of a week).

The mean schedule accuracy of the algorithm across the 11 operational schedules was 99.7% (SD, 0.1%; range, 99.7% to 99.9%), where schedule accuracy is calculated as the ratio of the sum over all surgical groups of shortfalls between each group's target and assigned OR time to the total time available on the schedule. The mean execution time for the 11 operational scenarios on a 650 MHz computer running Windows 98(SE) was 191 s (SD, 54 s; range, 124–293 s).

Applicability of Results to Other Hospitals

The accuracy of the results observed in other surgical suites will depend on a variety of factors, including the number of ORs that are available to be assigned, the duration of blocks, the number of surgical groups who have access to OR time, and the number and type of problem specific constraints (e.g., equipment availability or operational restrictions of the type "Dr. Smith must have an outpatient room on Mondays") (5). To explain this observation, we consider two hypothetical hospitals. One hospital has 10 staffed ORs each day of the workweek. The algorithm therefore has 50 (5 days × 10 ORs) blocks that it can assign in whole or in part to maximize

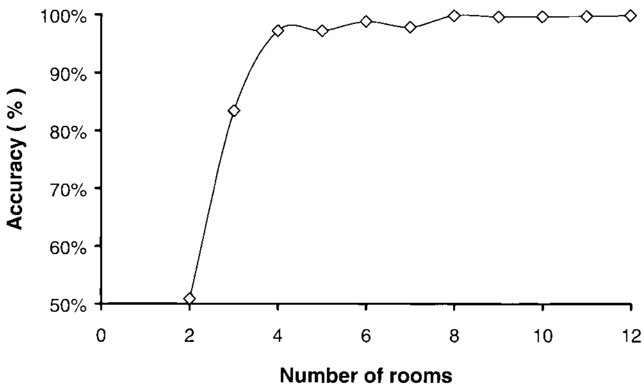

Figure 3. Schedule accuracy versus number of ORs available to be scheduled. This figure confirms that as the number of ORs decreases, the accuracy of the algorithm decreases. In this test, the model's performance falls off when the number of ORs to be scheduled is less than four.

schedule accuracy. This can be compared with another hospital with two staffed ORs each workday. In that instance, the algorithm has only 10 (5 days × 2 ORs) blocks that it can assign to maximize schedule accuracy. Because the smaller number of blocks provides the algorithm with less flexibility to generate different alternatives, the algorithm will be, in general, less accurate when there are fewer ORs.

To determine the extent to which results from Mt. Sinai are likely to be applicable to other facilities, we used an operational schedule produced for Mt. Sinai, but artificially reduced the number of ORs from 12 to 2. The minimum number of ORs that could be assigned to a particular surgical group on a given day or over a week was set to zero to ensure a feasible solution could be identified in all runs. Figure 3 shows the results of this analysis. The algorithm's accuracy exceeded 97% in runs in which 4 or more ORs were available.

Discussion

In this case study, we described a method to assign systematically blocks to surgical groups operating at a surgical suite. The method is appropriate for hospitals with limited OR budgets that must modify their master surgical schedules in response to changes in available funding, staff availability, or patient demand. Different algorithms will apply to hospitals at which OR time is always provided for all surgeons' patients (8) or for institutions that have the ability to use surgical resources to generate additional revenue.

Implementation of the automated methodology to create the master surgical schedule simplified the OR manager's task, because each entry in the proposed schedule was no longer "nit-picked." It is also our impression that conflict among surgeons and between surgeons and the OR manager was reduced because the schedule is produced in a consistent, unbiased, and identifiable manner. The algorithm requires <5 minutes of computer time on a personal computer, and as such is applicable in a variety of circumstances. For surgical suites with four or more ORs, it can achieve an accuracy exceeding 97%. In practice, the model has been found to have modest human resources requirements. A set of scenarios can usually be defined, input into the software, and evaluated in less than two hours.

Appendix

We define several parameters. Each of N_{group} surgical groups ($j = 0, 1, \ldots, N_{group}-1$) has a target allocation of OR time, t_j. There are seven different days of the week ($k = 0, 1, \ldots, 6$). There are N_{types} different types of ORs ($i = 0, 1, \ldots, N_{types}-1$). The parameter d_{ik} represents the hours that the i^{th} type of operating room is staffed on the k^{th} day of the week.

The total hours of OR time assigned to the j^{th} group equals

$$\sum_{i=0}^{N_{types}-1} \sum_{k=0}^{k=6} d_{ik} x_{ijk}$$

where x_{ijk} is the integer variable representing the number of ORs of type i assigned to group j on the k^{th}day of the week. The symbol

$$\sum_{i=0}^{N_{types}-1} \sum_{k=0}^{k=6} d_{ik} x_{ijk}$$

means the double sum across all room types (for $i = 0$ to $N_{types-1}$) and days (for $k = 0$ to 6) of all time assigned to service j ($d_{ik}x_{ijk}$). We define

$$s_j^-$$

to be a variable representing the under supply of OR time actually allocated or assigned to the j^{th} group relative to its target allocation t_j:

$$s_j^- = \max\left(0,\ t_j - \sum_{i=0}^{N_{types}-1} \sum_{k=0}^{k=6} d_{ik} x_{ijk}\right) \qquad (1)$$

The penalty associated with the under supply of OR hours to service j is

$$s_j^- / t_j.$$

The integer programming problem of equitably allocating OR time to surgical groups is to minimize the weighted sum of undersupplied OR hours,

$$\sum_{j=0}^{N_{groups}-1} s_j^- / t_j \qquad (2)$$

subject to equation (1) and the following four constraints.

First, the number of ORs of type i assigned to all groups on the k^{th} day must equal the total number of ORs of that type (a_{ik}):

$$\sum_{j=0}^{N_{group}-1} x_{ijk} = a_{ik}, \qquad (3)$$

for all i and k.

Second, the number of ORs of all types assigned to the j^{th} group on the k^{th} day of the week must be at least $LDAll_{jk}$ and at most $UDAll_{jk}$:

$$LDAll_{jk} \leq \sum_{i=0}^{N_{types}-1} x_{ijk} \leq UDAll_{jk}, \qquad (4)$$

for all j and k. For example, if the oral surgery group has three surgeons, the group must be assigned at least 0 and no more than 3 ORs each day.

Third, the number of ORs of the i^{th} type assigned to the j^{th} group on the k^{th} day must be at least $LDType_{ijk}$ and at most $UDType_{ijk}$:

$$LDType_{ijk} \leq x_{ijk} \leq UDAll_{ijk}, \qquad (5)$$

for all i, j, and k. For example, the gynecology group must have at least 0 and no more than 1 OR in the outpatient surgery suite every weekday.

Fourth, the number of ORs of the i^{th} type assigned to the j^{th} group each week must be at least $LWeek_{ij}$ and at most $UWeek_{ij}$:

$$LWeek_{ij} \leq \sum_{k=0}^{6} x_{ijk} \leq UWeek_{ij}, \qquad (6)$$

for all i and j. For example, the ophthalmology group must have at least 2 and at most 2 ORs in the outpatient surgery suite each week.

Equations (1) through (6) produce a master surgical schedule with a 1-wk horizon that can be extended to cover all weeks of the calendar month. This perfectly satisfies the objective that the schedule be similar from week to week. However, further decreases in the weighted average under supply of OR time to each group [Equation (2)] can often be achieved by relaxing this condition. A heuristic procedure calculates the change in the under supply of OR time to surgical groups [Equation (2)] by evaluating every possible substitution of one assigned block, of a particular type and on a particular day of the week, from one surgical group to another. A substitution is rejected if it violates one or more of the constraints given by Equations (3)–(6) or the additional constraint that no block can be assigned to more than two surgical groups. From this list of permissible substitutions, the change that would most decrease Equation (2) is made. The process is repeated until no further decrease in Equation (2) can be achieved.

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